USN

15MAT31

Third Semester B.E. Degree Examination, June/July 2018 Engineering Mathematics - III

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

Obtain the Fourier series for the function:

$$f(x) = \begin{cases} -\pi, & \pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$.

(08 Marks)

b. Obtain the half-range cosine series for the function $f(x) = (x-1)^2$, $0 \le x \le 1$. Hence deduce

that
$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$$

(08 Marks)

a. Find the Fourier series of the periodic function defined by $f(x) = 2x - x^2$ 0 < x < 3. (06 Marks) b. Show that the half range sine series for the function $f(x) = 2x - x^2$ in 0 < x < 1 is

$$\frac{8\ell^2}{\pi^3} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^3} \sin\left(\frac{2n+1}{\ell}\right) \pi x.$$

(05 Marks)

c. Express y as a Fourier series upto 1st harmonic given:

				1		
X	0	1	2	3	4	5
y	4	8	15	7	6	2

(05 Marks)

Module-2

Find the Fourier transform of

$$f(x) = \begin{cases} 1 - |x|, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$$

and hence deduce that
$$\int_{0}^{\infty} \frac{\sin^2 t}{t^2} dt = \frac{\pi}{2}.$$

(06 Marks)

Find the Fourier Sine and Cosine transforms of $f(x) = e^{-\alpha x}$, $\alpha > 0$.

(05 Marks)

c. Solve by using
$$z$$
 – transforms $y_{n+1} + \frac{1}{4}y_n = \left(\frac{1}{4}\right)^n$ $(n \ge 0), y_0 = 0$.

(05 Marks)



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OR

4 a. Find the Fourier transform of $f(x) = e^{-|x|}$.

(06 Marks)

b. Find the Z – transform of sin(3n + 5).

(05 Marks)

c. Find the inverse Z – transform of : (z-1)(z-2)

(05 Marks)

Module-3

5 a. Find the correlation coefficient and the equation of the line of regression for the following values of x and y. (06 Marks)

X	1	2	3	4	5
у	2	5	3	8	7

b. Find the equation of the best fitting straight line for the data:

(05 Marks)

X	0	1	2	3	4	5
У	9	8	24	28	26	20

c. Use Newton – Raphson method to find a real root of the equation $x \log_{10} x = 1.2$ (carry out 3 iterations).

OR

6 a. Obtain the lines of regression and hence find the coefficient of correlation for the data:

X	1	2	3	4	5	6	R
У	9	8	10	12	11	13	14

(06 Marks) (05 Marks)

b. Fit a second degree parabola to the following data:

		0			06
X	1	2	3	4	5
V	10	12	130	16	19

c. Use the Regula-Falsi method to find a real root of the equation $x^3 - 2x - 5 = 0$, correct to 3 decimal places. (05 Marks)

Module-4

- 7 a. Given Sin45° = 0.7071, Sin50° = 0.7660, Sin55° = 0.8192, Sin60° = 0.8660 find Sin57° using an appropriate interpolation formula. (06 Marks)
 - b. Construct the interpolation polynomial for the data given below using Newton's divided difference formula:

X	2	4	5	6	8	10
У	10	96	196	350	868	1746

(05 Marks)

c. Use Simpson's $\frac{1}{3}$ rd rule with 7 ordinates to evaluate $\int_{2}^{8} \frac{dx}{\log_{10} x}$. (05 Marks)



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OR

8 a. Given f(40) = 184, f(50) = 204, f(60) = 226, f(70) = 250, f(80) = 276, f(90) = 304, find f(38) using Newton's forward interpolation formula. (06 Marks)

b. Use Lagrange's interpolation formula to fit a polynomial for the data:

X	0	1	3	4
y T)-12	0	6	12

Hence estimate y at x = 2

(05 Marks)

c. Evaluate $\int_{0}^{1} \frac{x}{1+x^2} dx$ by Weddle's rule taking seven ordinates and hence find $\log_e 2$.

(05 Marks)

Module-5

- 9 a. Find the area between the parabolas $y^2 = 4x$ and $x^2 = 4y$ using Green's theorem in a plane.
 - b. Verify Stoke's theorem for the vector $\overrightarrow{F} = (x^2 + y^2)i 2xyj$ taken round the rectangle bounded by x = 0, x = a, y = 0, y = b.
 - c. Find the extremal of the functional : $\int_{x_1}^{x_2} [y' + x^2(y')^2] dx$. (05 Marks)

OB

- 10 a. Verify Green's theorem in a plane for $\oint_c (3x^2 8y^2) dx + (4y 6xy) dy$ where c is the boundary of the region enclosed by $y = \sqrt{x}$ and $y = x^2$. (06 Marks)
 - b. If $\overrightarrow{F} = 2xyi + yz^2j + xzk$ and S is the rectangular parallelopiped bounded by x = 0, y = 0, z = 0, x = 2, y = 1, z = 3 evaluate $\iint_S \overrightarrow{F} \cdot \overrightarrow{n} \, ds$. (05 Marks)
 - c. Find the geodesics on a surface given that the arc length on the surface is $S = \int_{x_1}^{x_2} \sqrt{x[1+(y')^2]} \, dx \, . \tag{05 Marks}$